Exploring Pascal's Triangle Northwest Iowa Math Teachers' Circle

(Based on notes by Tom Davis: http://www.geometer.org/mathcircles/pascal.pdf

1) Make a Pascal's Triangle on your own. (Presenter will describe basic idea.)

2) Investigate Pascal's triangle….what patterns do you find?

(Do not turn page until instructed.)

3) Perhaps you found some of these. Can you justify why they are true?

   a. All the numbers are positive. Why must this always be true?

   b. The numbers are symmetric about a vertical line through the top "1." Why?

   c. Look at lines of numbers parallel to the edge line of 1s.

      i. The line 1, 3, 6, 10, 15, 21, … are called triangle numbers. Why? Can you find a formula for the n^{th} triangular number?

      ii. What name should we have for the next line 1, 4, 10, 20, 35, …? Where do you they come from?

      iii. What could the line 1, 2, 3, 4, 5, … mean?

      iv. What could the line 1, 5, 15, 35, … mean?

d. What pattern do you get from summing rows? Why?

e. Sum along half diagonals, that is halfway between a row and a line in the triangle. What numbers do you get? Can you explain why these numbers arise.

4) Expand algebraically:

a. $(L + R)^0 =$

b. $(L + R)^1 =$

c. $(L + R)^2 =$

d. $(L + R)^3 =$

e. $(L + R)^4 =$

f. $(L + R)^5 =$

g. The coefficients of a particular LR term, e.g., L^2R^1 should match the number in the triangle from taking 2 left steps and 1 right. Why does this work?

5) Let's say you want to know what number is the 6^{th} number in the 10^{th} row. Can you find it without working through all the rows above it, or is there a formula? With this goal in mind

   a. Presenter explains notation and idea of "n choose k". How many ways can you choose k items from a set of size n, e.g., {A, B, C} then "3 choose 2" = 3 because AB, BC, and AC are the three possibilities. What is "6 choose 3"?

   b. Suppose we have {A, B, C, D, E, F, G} and we want "7 choose 3". Note that some sets include "A" and some don't. How many ways are there to choose a set with A in it, and how many ways are there to choose a set without A in it. This means that "7 choose 3" is equal to:

   c. In general, "n choose k" must be equal to:

   d. Notice that that is exactly how Pascal's triangle is constructed. Can you think of a way to answer the counting of "n choose k" question? That will necessarily be the answer to the n^{th} row, k^{th} number question.