“Inspiration is needed in geometry, just as much as in poetry.” -Alexander Pushkin

A Warm Up Problem

1. Suppose the following star-like diagram has 5-fold symmetry. If the area of the whole star is 20 and the area of the smaller kite shaped piece is 8, what is the area of the inside pentagon? Can you find more than one way of solving this?

Introduction to Geometry on a Sphere

In your group you have a Lénart Sphere and some ribbon. You may use these tools to help you study some geometric concepts on the sphere.

1. In the plane, lines are straight. Why? What exactly does that mean?

2. What is “straight” on a sphere?

3. What are some differences between lines in the plane and lines on a sphere?
Triangles on a Sphere

Now that we know a bit about lines on a sphere, let’s look at some triangles.

1. Use the markers and the line tool to make a triangle on the sphere. Do you notice anything different?

2. Do any three non-collinear points determine a triangle?

Similar Triangles on a Sphere

We will now look at few examples of triangles on the sphere. This will mostly be an exploration using the Lénart spheres, but try to make notes and answer the questions as best you can.

1. Make a triangle in following way. Use the equator as a line, and make a point at the north pole. Draw a straight line down from the north pole to the equator. Call the point at the north pole \( A \) and the point on the equator \( B \). What is \( m\angle B \)?

2. Now mark a new point \( C \) on the equator and draw a line down from \( A \) to \( C \). What is \( m\angle C \)?
   What kind of triangle is \( \triangle ABC \)?

3. Can you make a triangle with three right angles?

4. Does the area formula \( A = \frac{1}{2}Bh \) work on a sphere?

5. Try to make two similar triangles on your sphere. Can you?
Area on a Sphere

The last concept we will consider is that of area on a sphere. Assume for the rest of this section that your sphere has a radius of 1 unit. (Actually the radius is 10 cm, so really we are working in decimeters as our base unit.)

1. What is the area of the entire sphere?

2. Draw two lines from the north pole to the south pole. This will create a wedge region called a lune. Use the protractor to measure the angle of your lune (in radians). What is the area of your lune? What is a general area formula for lunes?

3. Draw another line on the sphere that cuts across your lune to make a triangle close to the north pole. Label the points as $A$, $B$, and $C$, and write in the angles as $a$, $b$, and $c$ (angles in radians).

4. This is the capstone activity of the evening. The goal now is to find a formula for the area of a triangle on the sphere. Use the fact that you know that total area of the sphere and the area of the three lunes to derive the triangle area formula. (The idea from the warm-up problem is key here.)
Classroom Connections

1. Are there any connections between the math we’ve done here and what you teach in your classroom?

2. Is there an area of your curriculum that might benefit from something we’ve done here tonight?

3. How would you have improved the activity for this context or for your classroom?

I’ve included below a few standards from the CCSS that we have touched on in one way or another tonight.

- Standards for Mathematical Practice:
  - 1. Make sense of problems and persevere in solving them.
  - 2. Reason abstractly and quantitatively.
  - 3. Construct viable arguments and critique the reasoning of others.
  - 4. Model with mathematics.
  - 5. Use appropriate tools strategically.

- Geometry Standards:
  - The CCSS geometry standards are mostly specific to Euclidean Geometry, so this activity fits well in the general theme/scope of the standards, but doesn’t explicitly hit them. The activity gets at geometric concepts but in a different way. You can decide whether it might be appropriate to do something like this in a HS geometry course or not.